Bilinear Programming Approach for Dynamic Output Feedback Constrained Control of Linear Systems subject to Persistent Disturbances

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Abstract

The objective is to extend previous results obtained in the context of Static Output Feedback (SOF) control to the design Dynamic Output Feedback (DOF) control laws for linear systems subject to persistent disturbances and to state and control constraints (such as amplitude and rate limitations), by dealing with together the computation of a stabilizing control law and the determination of a Δ-invariant polyhedral set with a related ultimately bounded UB-set, to guarantee the respect of state and control constraints.

To this end, the candidate should have a background on linear systems dynamical analysis and control, including the understanding of robust control, and will have to deeply study and comprehend: i) the formulation of the considered constrained control problems; ii) the development and meaning of Δ-invariance concept and of the new proposed algebraic conditions, which also contains underlying important basics for the proposal; and ii) the design and implementation issues of the bilinear optimization design strategy. In particular, numerical examples related to real control applications will be also considered.

Keywords: Dynamic output feedback; constrained control; persistent disturbances; Δ-invariance; Bilinear programming.

1 Introduction

Set invariance has been recognized as a powerful concept and has been extensively exploited to treat various constrained control problems where it is necessary to deal with practical constraints such as amplitude limits applied to control, state or output variables If a dynamical system is subject to persistent (*i.e.:* amplitude bounded) disturbances, the so-called Δ-invariance property guarantees that any trajectory starting from a set inside the state space will remain in this set and, possibly, will be ultimately bounded in some of its subsets.

Although ellipsoidal sets have been more usually considered in the literature, mainly due to the possibility of using LMI based algorithm for analysis and synthesis of constrained control systems, polyhedral sets have been receiving a great deal of attention due to the fact that their shape better fits to the amplitude bounded constraints. In special, algebraic conditions describing the Δ-invariance property of polyhedral sets with respect to time-invariant and uncertain linear systems subject to persistent disturbances have been developed and used to treat related control problems. For instance, by considering a given polyhedral set of state constraints and amplitude bounded control inputs, then a linearly constrained control problem consisting of encountering a linear state-feedback control law that guarantees the Δ-invariance of the set of state constraints while guaranteeing that only admissible control inputs are applied, can be solved through a Linear Programming (LP) optimization problem where the closed-loop Δ-invariance relations appear as part of the LP constraints However, when the set of state constraints cannot be made Δ-invariant by a linear or even a non-linear feedback control law, solutions to the related problems become less direct. In such cases, the interest becomes to make Δ-invariant by feedback another set, possibly a polyhedral one and as big as possible, contained in the set of state constraints. Even for the synthesis of linear control laws, the use of Δ-invariance relations to render controlled invariant an a-priori unknown set characterizes non-linear constraints, hence non-convex, with respect to the involved matrix variables. Thus, linear programming or convex optimization methods cannot be directly applied.

Furthermore, solving the above mentioned constrained control problem becomes still more involving when a an Output Feedback control law (static or dynamic) and the determination of an associate ultimately bounded (UB-) set are sought out together. This particular constrained control has been considered in the Doctorate thesis of Stephanie Loi Brião (PPGEAS/UFSC). To this end, a new set of algebraic conditions that guarantees the Δ-invariance property and contractivity of an uttermost set with an associated innermost UB-set to which the trajectories converge finite-time has already been proposed. Thus, from these algebraic relations, a bilinear approach was formulated to solve an associated constrained control problem using a SOF control law, allowing to deal with the optimization of the size of the two associated sets by considering an objective function that weighs the maximization of the uttermost set while minimizing the relative size of innermost one. Thus, the KNITRO nonlinear solver was used to tackle the bilinear approach.